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On the Interaction of Mesotrons and Nuclei

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THE formalism of mesotron field theories was first developed in analogy either with the method of classical electrodynamics that involves the determination of fields produced by pregiven sources, or with the perturbation technique of quantum electrodynamics. This program not only has led to predictions in contradiction with experiment (the singular nature of nuclear forces, and large cross sections for mesotron scattering by nuclei) but is intrinsically inconsistent, for the methods of electrodynamics, involving the neglect of the reaction of field on source, and based on the small magnitude of the coupling between charges and the electromagnetic field, are here quite unjustified. This inconsistency is rendered far sharper by the charge- and spin-dependent couplings of the mesotron theory. These difficulties have led to two apparently unrelated suggestions for modifying the formal treatment of mesotron theory.

Thus Heisenberg has attempted a more complete solution of the mesotron equations by including, within the framework of a classical theory, the large reaction of the field on the emitting source. He treated the problem of a neutral vector field interacting with a fixed, spatially extended source to which is attached a classical spin. His calculations showed that the inertia of the spin, arising from the reaction of its proper field, would greatly reduce the spin-dependent mesotron scattering. A relativistic variant of this procedure has been elaborated by Bhabha, who based his work on Dirac's classical electron model. Those terms in the field reaction on the source which become singular as a, the spatial extension of the source, vanishes, are discarded and the finite terms are evaluated in the limit of point coupling. However, it is precisely these singular terms that are essential for Heisenberg's explanation of the small mesotron scattering. Furthermore, quite apart from the serious question of whether this method affords a classical description of elementary particles, it is so essentially restricted to the classical domain that it provides no basis for a correspondence treatment of the actual quantum-mechanical problems encountered in a discussion of the scattering of charged mesotrons and of nuclear forces.

For this reason Bhabha himself and Heitler suggested the alternative theory of proton isobars. They observed that, from the point of view of perturbation theory, the large scattering arising from charge- and spin-dependent couplings is a consequence of the prohibition of certain intermediate states in the scattering process by the conservation laws for charge and angular momentum. They therefore proposed a modification of the assumptions of mesotron theory by postulating the existence of slightly more massive nuclear particles with arbitrary integral charge and half-integral spin. By a suitable choice of the excitation energy of the lowest isobars—high enough to make them escape ready detection—the scattering cross section could be reduced sufficiently to avoid conflict with experiment.

The connection between these two sets of ideas lies in the circumstance that a field strongly coupled to a charge or spin-dependent source will itself possess states in which charge or angular momentum is bound to the source, with the energy of the system increasing quadratically in its dependence on the total charge or angular momentum. This has been shown in detail by Wentzel for the charged scalar field. Wentzel used a lattice space to achieve convergence and considered only the limit of strong coupling. Despite the fact that his theory predicted isobars, his calculations indicated an inacceptably large value for the mesotron scattering cross section.

The evidence of nuclear forces shows, however, that the mesotron coupling is spin dependent, and indicates that the coupling constant itself is not very large. In order to see how these facts would modify Wentzel's conclusions, we have investigated a class of problems that throws some light on the earlier work and that also offers promise of a limited but consistent description of the mesotron, in agreement with experience: nuclear forces on the one hand, and, on the other, the small scattering, zero spin, and highly multiple production observed for mesotrons in cosmic rays. We have, in part, generalized Heisenberg's treatment, and considered the classical problem of the coupling of neutral and charged, scalar and pseudo-scalar mesotrons to an extended, spatially fixed source. These problems are all rigorously soluble, for all values of the coupling constant and source size. In addition, we have treated Wentzel's quantum problem of the charged scalar field,[1] using an extended source instead of a lattice space, in the limit where the coupling constant g is large, $\gamma = g^2/\hbar c \gg 1$, and have made the analogous calculation for the neutral pseudo-scalar in the corresponding limit $\gamma \gg \kappa a$ (here $\kappa = \mu c/\hbar$ and μ is the mesotron mass). The problem of the charged pseudo-scalar has not been solved quantum-mechanically even in this limit, nor has any quantum solution been found for intermediate values of γ, or $\gamma/\kappa a$; nor have the revisions in nuclear forces been calculated in detail.

Nevertheless, we believe our results to be of some interest. The classical solutions for the pseudo-scalar always give states of bound angular momentum if $\gamma/\kappa a$ is not too small; the energy of these states depends on the total angular momentum J, as

$$\tfrac{3}{2}(\kappa a/\gamma)\mu c^2(J^2 - \tfrac{1}{4}).$$

Here a is defined by

$$a^{-1} = \int d\tau K(\mathbf{r})\,|\mathbf{r} - \mathbf{r}'|^{-1} K(\mathbf{r}')d\tau'$$

and $K(\mathbf{r})$ is the source function. The symmetrical scalar and charged scalar theories give solutions with bound charge, the former only if $2\gamma > 1$; and here the mass of the normal state of an isobar, for $\gamma \gg 1$, depends on the charge Q as $(\mu/\gamma)(Q - \tfrac{1}{2})^2$. For the symmetrical pseudo-scalar the condition for the existence of bound spin and charge is $2\gamma > 3\kappa a$. The quantum solutions for $\gamma \gg 1$ (charged scalar) and $\gamma \gg \kappa a$ (neutral pseudo-scalar) agree in these conclusions.[2] This agreement confirms the *a priori* expectation that, for large enough coupling, the quantum fluctuations of the source will be negligible compared to the reaction of the source to the field.

Classical scattering formulae, applicable when the proton recoil is negligible, give for all pseudo-scalar theories a scattering vanishing with a^2: this is a direct consequence of the fact that, in these theories, only p states are coupled to the source. Thus, for $\gamma \gg \kappa a$, and a mesotron momentum $p < \hbar/a$, the scattering cross section for a charged mesotron on the symmetrical pseudo-scalar theory is

$$d\sigma = \tfrac{3}{5}a^2(pc/E)^4(1 + 2\cos^2\vartheta)d\Omega, \tag{1}$$

whereas for a charged scalar with $\gamma \gg 1$, $a \rightarrow 0$, it is just

$$d\sigma = (\hbar c/E)^2 d\Omega. \tag{2}$$

The results of the quantum solution agree with (2) for the charged scalar, and are of the same form as (1) for the neutral pseudo-scalar.

It is thus clear that pseudo-scalar theories can give a scattering small enough to agree with that observed, but that scalar theories could, at most, do so with a choice of γ far too small to account for nuclear forces. In fact, the experimental scattering results demand a value of a of the order of the proton Compton wave-length $\hbar/Mc$, or possibly slightly smaller. Indeed, this length marks the extreme limit of validity of the methods we are using, and of the classical localizability of the source.

With this small value of a, and a value $\gamma \sim \frac{1}{10}$, which is derived from the magnitude of nuclear forces in singlet states, the spin and charge isobars will have an excitation only somewhat smaller than the rest energy of the mesotron.

This small value of γ would at first sight seem to be inconsistent with the high multiplicity of mesotron production in high energy nuclear collisions; but for pseudo-scalar theories this is not so. We have made an admittedly crude estimate of this multiplicity by calculating classically the excitation of the mesotron field when a proton's velocity and spin are suddenly

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[1] Julian Schwinger, to be published soon.

[2] G. Wentzel (Helv. Phys. Acta **13**, 269 (1940)) gives $\gamma\kappa a \gg 1$ as the condition for the validity of his solution and, for the isobar separation $(\mu/\gamma\kappa a)(Q - \frac{1}{2})^2$. In both results $\gamma\kappa a$ must be replaced by γ. Wentzel's conclusions were obtained by overlooking the contribution to the isobar energy of order g^{-1} in the expansion of the Hamiltonian in descending powers of g. For a detailed discussion see reference 1.

altered, and then limiting the actual spectrum to mesotrons of total energy equal to or lower than the energy loss ΔE of the proton. In this way one finds a multiplicity $N \sim \gamma^{\frac{1}{3}}(\Delta E/\mu c^2)^{\frac{2}{3}}$, with a value 10 for $\gamma = \frac{1}{10}$, and an energy loss of 10^{10} volts. Under these conditions, and with a comparable value of γ, a scalar theory would give practically no multiple production.

It will be observed that we have used a value of γ given by the perturbation theoretic evaluation of nuclear forces. This is because, for singlet states, our theories still give forces of range $\hbar/\mu c$, which have an effective depth of the order $\gamma \mu c^2$, and which behave like g^2/a for $\gamma < a$. The detailed radial dependence, and in particular the form of the tensor forces, remains to be investigated.

In conclusion, we should like again to emphasize that our present methods involve, in their physical content, only a rather more complete effort to take into account the reaction of the source to the mesotron field than either Bhabha's classical methods or the *a priori* postulation of isobars afforded. But it would seem that these methods are sufficient to decide definitely in favor of a pseudo-scalar, rather than a scalar or vector, field and to fix roughly the values of the coupling constant and source size needed to make the model definite.